VASAVI COLLEGE OF ENGINEERING (AUTONOMOUS), HYDERABAD
Accredited by NAAC with A++ Grade
B.E. III-Semester Supplementary Examinations, August-2023
Complex Variables (OE-I)

Time: **3 hours**

Max. Marks: **60**

*Note: Answer all questions from **Part-A** and any **FIVE** from **Part-B***

Part-A (10 × 2 = 20 Marks)

Q. No.	Stem of the question	M	L	CO	PO
1.	Define analytic function and give an example.	2	1	1	1,12
2.	Define harmonic function and give an example.	2	1	1	1,12
3.	State generalized Cauchy's integral formula.	2	1	2	1,12
4.	Evaluate $\oint_C \frac{e^z}{z^2+1} dz$, where C is the circle $|z| = \frac{1}{2}$.	2	2	2	1,2
5.	Define Isolated singularity and removable singularity.	2	1	3	1,12
6.	Define Pole and Essential singularity.	2	1	3	1,12
7.	Define residue. What is the residue if "a" is a pole of order n.	2	1	4	1,12
8.	State Cauchy's residue theorem.	2	1	4	1,12
9.	State sufficient condition for a function to be analytic.	2	1	1	1,12
10.	Find $\oint_C \frac{dz}{(z-a)^n} dz, n = 2, 3, 4, \ldots$, where C is a closed curve containing the point $z = a$.	2	2	2	1,2

Part-B (5×8 = 40 Marks)

11. a)	State and prove necessary condition for a function to be analytic.	4	2	1	1,12
b)	Find the analytic function, whose real part is $e^{-x}(x \sin y - y \cos y)$.	4	3	1	1,12
12. a)	State and prove Cauchy's theorem.	4	2	2	1,12
b)	Evaluate $\oint_C \frac{z}{z^2-3z+2} dz$, where C is $|z - 2| = \frac{1}{2}$ using Cauchy's integral formula.	4	3	2	1,2
13. a)	Find Taylor's expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point $z = i$.	4	3	3	1,12
b)	Find the nature and location of singularities of the following functions: (i) $\frac{z-\sin z}{z^2}$ (ii) $(z + 1) \sin \frac{1}{z-2}$	4	4	3	1,2

			M	L	CO	PO
14.	a)	Find the sum of the residues of $f(z) = \dfrac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$.	4	2	4	1,12
	b)	Evaluate $\oint_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}\, dz$, where C is the circle $|z| = 3$ using residue theorem.	4	3	4	1,12
15.	a)	Show that $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{(x^2+y^2)}$, $z \neq 0$ and $f(0) = 0$ is not analytic at $z = 0$, although C-R equations are satisfied at the origin.	4	3	1	1,12
	b)	State and prove Cauchy's Integral formula.	4	2	2	1,12
16.	a)	Find the Laurent's series expansion of $f(z) = \dfrac{1}{(z-1)(z-2)}$ in the region $1 < |z| < 2$.	4	3	3	1,12
	b)	Evaluate $\oint_C \dfrac{3z^2+z+1}{(z^2-1)(z+3)}\, dz$, where C is the circle $|z| = 2$ using residue theorem.	4	3	4	1,12
17.		Answer any *two* of the following:				
	a)	Find the harmonic conjugate function of $e^{-2xy} \sin(x^2 - y^2)$.	4	2	1	1,12
	b)	Evaluate $\oint_C \dfrac{\sin^2 z}{(z - \pi/6)^3}\, dz$, where C is the circle $|z| = 1$ using Cauchy's Integral formula.	4	3	2	1,12
	c)	Find the Laurent's expansion of $f(z) = \dfrac{7z-2}{(z+1)z(z-2)}$ in the region $1 < z+1 < 3$.	4	3	3	1,12

M : Marks;   L: Bloom's Taxonomy Level;   CO; Course Outcome;   PO: Programme Outcome

i)	Blooms Taxonomy Level – 1	20%
ii)	Blooms Taxonomy Level – 2	30%
iii)	Blooms Taxonomy Level – 3 & 4	50%

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